Exercise 308

The rabbit population on a game reserve doubles every 6 months. Suppose there were 120 rabbits initially.

a. Use the exponential function $P = P_0 a^t$ to determine the growth rate constant a. Round to four decimal places.

b. Use the function in part a. to determine approximately how long it takes for the rabbit population to reach 3500.

Solution

Part (a)

Use the fact that the rabbit population doubles every 6 months to determine a.

$$P(t) = P_0 a^t$$

$$240 = 120a^6$$

Divide both sides by 120.

$$2 = a^6$$

Take the sixth root of both sides to get a.

$$a = \sqrt[6]{2} \approx 1.225$$

Part (b)

Plug in 3500 for $P(t)$, 120 for P_0, and the result for a from part (a).

$$P(t) = P_0 a^t$$

$$3500 = 120(1.225)^t$$

Divide both sides by 120.

$$\frac{175}{6} = 1.225^t$$

Take the natural logarithm of both sides.

$$\ln \frac{175}{6} = \ln 1.225^t$$

Use the property of logarithms that allows the exponent of the argument to be brought down in front.

$$\ln \frac{175}{6} = t \ln 1.225$$

Divide both sides by $\ln 1.225$ to solve for t.

$$t = \frac{\ln \frac{175}{6}}{\ln 1.225} \approx 29.2$$

Therefore, it will take about 29 months for the rabbit population to reach 3500.